

# Isoscalar Giant Dipole Resonance and Nuclear Matter Incompressibility Coefficient

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We present results of microscopic calculations of the strength function,  $S(E)$ , and  $\alpha$ -particle excitation cross sections  $\sigma(E)$  for the isoscalar giant dipole resonance (ISGDR). An accurate and a general method to eliminate the contributions of spurious state mixing is presented and used in the calculations. Our results provide a resolution to the long standing problem that the nuclear matter incompressibility coefficient,  $K$ , deduced from  $\sigma(E)$  data for the ISGDR is significantly smaller than that deduced from data for the isoscalar giant monopole resonance (ISGMR).

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Studies of compression modes of nuclei are of particular interest since their strength distributions,  $S(E)$ , are sensitive to the value of the nuclear matter incompressibility coefficient,  $K$  [1,2]. Over the last two decades, a significant amount of experimental work was carried out to identify strength distributions of the isoscalar giant monopole resonance (ISGMR) in nuclei [3]. At present, Hartree-Fock (HF) based random-phase-approximation (RPA) calculations for the ISGMR reproduce the experimental data for effective interactions associated with incompressibility  $K = 210 \pm 20$  MeV [4].

The study of the isoscalar giant dipole resonance (ISGDR) is very important since this compression mode provides an independent source of information on  $K$ . Early experimental attempts to identify the ISGDR in  $^{208}\text{Pb}$  resulted with a value of  $E_1 \sim 21$  MeV for the centroid energy [5,6]. Similar result for  $E_1$  in  $^{208}\text{Pb}$  was obtained in recent experiments [7]. Very recent and more accurate data on the ISGDR obtained for a wide range of nuclei [8] seems to indicate that the experimental values for  $E_1$  are smaller than the corresponding HF-RPA results by 3 – 5 MeV.

It was first pointed out in Ref. [9] that corresponding HF-RPA results for  $E_1$ , obtained with interactions adjusted to reproduce the ISGMR data, are higher than the experimental value by more than 3 MeV and thus this discrepancy between theory and experiment raises doubts concerning the unambiguous extraction of  $K$  from energies of compression modes (see also Ref. [10]).

In this work we address this discrepancy between theory and experiment by examining the relation between  $S(E)$  and the excitation cross section  $\sigma(E)$  of the ISGDR, obtained by  $\alpha$ -scattering. We emphasize that it is quite common in theoretical work on giant resonance to calculate  $S(E)$  for a certain scattering operator  $F$  whereas in the analysis of experimental data of  $\sigma(E)$  one carries out distorted-wave-Born-approximation (DWBA) calculations with a certain transition potential. Here we present results of accurate microscopic calculations for  $S(E)$  and for  $\sigma(E)$  with the folding model (FM) DWBA

with transition densities  $\rho_t(\mathbf{r})$  obtained from HF-RPA calculations. We provide a simple explanation for the discrepancy between theory and experiment concerning the ISGDR and suggest further experiments.

We also present an accurate projection method to eliminate the contributions of the spurious state mixing (SSM) to  $S(E)$  and  $\rho_t(\mathbf{r})$  of the ISGDR obtained in HF-RPA calculations for an operator  $F$ . The method, which is based on the replacement of  $F$  with a properly modified operator  $F_\eta$  in the calculation of  $S(E)$  and  $\rho_t(\mathbf{r})$ , is quite general and applicable for any  $F$  and for any numerical method used in carrying out the RPA calculation, such as configuration space RPA, coordinate space (continuum and discretized) RPA and with and without the addition of smearing. We note that this method was recently used in the continuum RPA calculation of  $S(E)$  in Ref. [11]. Some preliminary results of our calculation were given in Ref. [12].

In self-consistent HF-RPA calculation one starts by adopting specific effective nucleon-nucleon interaction,  $V_{12}$ , carries out the HF calculation for the ground state of the nucleus and then solves the RPA equation using the particle-hole (p-h) interaction  $V_{ph}$  which corresponds to  $V_{12}$ . The RPA Green's function  $G$  [13,14] is obtained from

$$G = G_0(1 + V_{ph}G_0)^{-1}, \quad (1)$$

where  $G_0$  is the free p-h Green's function. For

$$F = \sum_{i=1}^A f(\mathbf{r}_i), \quad (2)$$

the strength function and transition density are given by

$$S(E) = \sum_n |\langle 0|F|n \rangle|^2 \delta(E - E_n) = \frac{1}{\pi} \text{Im} [\text{Tr}(fGf)], \quad (3)$$

$$\rho_t(\mathbf{r}, E) = \frac{1}{\sqrt{S(E)}} \int f(\mathbf{r}') \left[ \frac{1}{\pi} \text{Im} G(\mathbf{r}', \mathbf{r}, E) \right] d\mathbf{r}'. \quad (4)$$

Note that the definition (4) is consistent with

$$S(E) = \left| \int \rho_t(\mathbf{r}, E) f(\mathbf{r}) d\mathbf{r} \right|^2. \quad (5)$$

In fully self-consistent HF-RPA calculations, the spurious state (associated with the center of mass motion)  $T = 0, L = 1$  appears at  $E = 0$  and no SSM in the ISGDR occurs. However, although not always stated in the literature, actual implementations of HF-RPA are not fully self-consistent. One usually makes the following approximations: (i) neglecting the two-body Coulomb and spin-orbit interactions in  $V_{ph}$ , (ii) approximating momentum parts in  $V_{ph}$ , (iii) limiting the p-h space in a discretized calculation by a cut-off energy  $E_{ph}^{max}$ , (iv) introducing a smearing parameter (i.e., a Lorentzian with  $\Gamma/2$ ) and (v) some numerical approximations. Although the effect of these approximations on the centroid energies of giant resonances is small (less than 1 MeV), the effect on the ISGDR is quite serious since each of these approximations introduces a SSM in the ISGDR.

The first serious attempt to correct for the effect of the SSM on  $S(E)$  and  $\rho_t$ , associated with  $f(\mathbf{r}) = f_3(\mathbf{r}) = r^3 Y_{1M}(\Omega)$ , was presented in Ref. [15]. However, this method is not accurate and leads to strong reduction in the ISGDR strength at low energies. In Refs. [16,17,9], the effect of the SSM on  $S(E)$  was ignored and was only considered with regard to the energy weighted sum rule (EWSR) and the derivation of the collective transition density. In a very recent configuration space RPA calculation for the ISGDR [10] a method for accounting for the SSM was implemented for  $f_3(\mathbf{r})$ . However, the results for  $S(E)$  and  $\rho_t$  depends on how the smearing is implemented (not described in the paper).

Let us consider scattering operators, Eq. (2), with

$$f(\mathbf{r}) = f(r) Y_{1M}(\Omega), \quad f_1(\mathbf{r}) = r Y_{1M}(\Omega), \quad (6)$$

and write  $\frac{1}{\pi} \text{Im}G$  as sum of separable terms

$$R(\mathbf{r}', \mathbf{r}, \omega) = \frac{1}{\pi} \text{Im}G(\mathbf{r}', \mathbf{r}, \omega) = \sum_a \rho_a(\mathbf{r}) \rho_a(\mathbf{r}'). \quad (7)$$

Note that  $\rho_a$  includes the energy dependence and is written as

$$\rho_a(\mathbf{r}) = \rho_{a3}(\mathbf{r}) + \rho_{a1}(\mathbf{r}), \quad (8)$$

where  $\rho_{a1}(\mathbf{r})$  is due to SSM and  $\rho_{a3}$ , associated with the ISGDR, fulfills the center of mass condition (for all  $a$ )

$$\langle f_1 \rho_{a3} \rangle = \int f_1(\mathbf{r}) \rho_{a3}(\mathbf{r}) d\mathbf{r} = 0. \quad (9)$$

From (7) and (8) we have in an obvious notation

$$R = R_{33} + R_{31} + R_{13} + R_{11}, \quad (10)$$

with  $R_{ij} = \sum \rho_{ai}(\mathbf{r}) \rho_{aj}(\mathbf{r}')$ , and the required  $S(E)$  and  $\rho_t$  can be obtained from  $R_{33}$  using (4) and (5) with  $f$ .

Since  $R_{33}$  is not known we look for a projection operator that projects out  $\rho_{a1}(\mathbf{r})$ ,

$$F_\eta = \sum_{i=1}^A f_\eta(\mathbf{r}_i) = F - \eta F_1, \quad (11)$$

with  $f_\eta = f - \eta f_1$ . Using (9) and (10) we have

$$S_\eta(E) = \langle f_\eta R f_\eta \rangle = \langle f R_{33} f \rangle + 2 \langle f R_{31} f_\eta \rangle + \langle f_\eta R_{11} f_\eta \rangle. \quad (12)$$

Note that  $\langle f_\eta R_{11} f_\eta \rangle$  is minimum for

$$\eta = \langle f R_{11} f_1 \rangle / \langle f_1 R_{11} f_1 \rangle = \sum \langle f \rho_{a1} \rangle \langle f_1 \rho_{a1} \rangle / \sum \langle f_1 \rho_{a1} \rangle^2, \quad (13)$$

and for the last two terms in (12) to vanish we must have

$$\langle f \rho_{a1} \rangle = \eta \langle f_1 \rho_{a1} \rangle, \quad \text{for all } a. \quad (14)$$

The condition (14) holds in case (7) has only one term, such as in a discretized calculation without smearing or in a configuration space calculation, and/or in case all  $\rho_{a1}(\mathbf{r})$  are proportional to the same transition density, the coherent spurious states transition density [18]

$$\rho_{a1}(\mathbf{r}) = \alpha_a \rho_{ss}(\mathbf{r}) = \alpha_a \frac{\partial \rho_0}{\partial r} Y_{1M}(\Omega), \quad (15)$$

where  $\rho_0$  is the ground state density of the nucleus. The value of  $\eta$  associated with  $\rho_{ss}$  is then given by

$$\eta = \langle f \rho_{ss} \rangle / \langle f_1 \rho_{ss} \rangle. \quad (16)$$

To determine  $\rho_t$  for the ISGDR we first use (4), (7), (8), (9) and (14) with  $F_\eta$  and obtain

$$\rho_\eta(\mathbf{r}) = \frac{1}{\sqrt{S_\eta(E)}} \sum c_a [\rho_{a3}(\mathbf{r}) + \rho_{a1}(\mathbf{r})], \quad (17)$$

with  $c_a = \langle f_\eta \rho_{a3} \rangle$ . To project out the spurious term from (17) we make use of (9) with  $\rho_{a1} = \alpha_a \rho_{ss}$  and obtained

$$\rho_t(\mathbf{r}) = \rho_\eta(\mathbf{r}) - \alpha \rho_{ss}, \quad \alpha = \langle f_1 \rho_\eta \rangle / \langle f_1 \rho_{ss} \rangle. \quad (18)$$

It is important to emphasize that using  $f_\eta$  in (5) with  $\rho_t(\mathbf{r})$  from (18), one obtains the required  $S_\eta(E)$ . This is due to the fact that the averaging process in (17) was carried out using  $F_\eta$  and not  $F$ . Use of  $F$  in (4) may produce erroneous results for  $\rho_t(\mathbf{r})$  in (18) in case there are several terms in (7).

We now limit our discussion to the operator  $F_3 = \sum_{i=1}^A f_3(\mathbf{r}_i)$ , where we have for (15)

$$\eta = \frac{5}{3} \langle r^2 \rangle. \quad (19)$$

We note that the values of  $\eta$  obtained for  $\rho_a$  associated with single p-h transitions in the  $1\hbar\omega$  region and the RPA results for the spurious state  $\rho_t$ , differ from that of (19) by less than 20% [19].

We have carried out numerical calculations for the  $S(E)$ ,  $\rho_t(\mathbf{r})$  and  $\sigma(E)$  within the FM-DWBA-HF-RPA theory. We used the SL1 Skyrme interaction [20], which is associated with  $K = 230$  MeV, and carried out HF calculations using a spherical box of  $R \geq 25$  fm. For the RPA calculations we used the Green's function approach with mesh size  $\Delta r = 0.3$  fm and p-h maximum energy of  $E_{ph}^{max} = 150$  MeV (we include particle states with principle quantum number up to 12), since it is well-known that in order to extract accurate  $\rho_t(\mathbf{r})$ ,  $E_{ph}^{max}$  should be much larger than the value required ( $E_{ph}^{max} \sim 50$  MeV) to recover EWSR. Since in our calculation we also neglected the two-body coulomb and spin-orbit interactions, the spurious state energies differ from 0 by a few MeV. We therefore renormalized the strength of the  $V_{ph}$  by a factor (0.99 and 0.974 for  $^{116}\text{Sn}$  and  $^{208}\text{Pb}$ , respectively), to place the spurious state at  $E=0.2$  MeV. We have included a Lorentzian smearing ( $\Gamma/2 = 1$  MeV) and corrected for the SSM as described above. We carried out the FM-DWBA calculation for  $\sigma(E)$  using a density dependent Gaussian nucleon- $\alpha$  interaction with parameters adjusted to reproduce the elastic cross section, with  $\rho_0$  and  $\rho_t$  from HF-RPA, see Ref. [23].

Using the operator  $f = r^2$  for the ISGMR we calculated the corresponding  $S(E)$ , for  $E$  up to 60 MeV. We recover 100% of the corresponding EWSR and obtained the values of 17.09 and 14.48 MeV for the centroid energy of the ISGMR in  $^{116}\text{Sn}$  and  $^{208}\text{Pb}$ , respectively. The corresponding recent experimental values are  $16.07 \pm 0.12$  and  $14.17 \pm 0.28$  MeV, respectively [21].

In Figure 1, we present the results of the  $ES_3(E)$  and  $ES_1(E)$  for  $^{116}\text{Sn}$ , obtained by using Eq. (3) with  $f_3$  and  $f_1$ , respectively. Note that from (9) and (10)  $S_1(E) = \langle f_1 R_{11} f_1 \rangle = \sum \langle f_1 \rho_{a1} \rangle^2$ , provides a measure to the contribution of the SSM to  $S_3(E)$ . The large contribution from the Lorentzian tail of the spurious state is clearly seen in the figure. Also shown in Figure 1, is the ratio  $\langle f_3 R f_1 \rangle / \langle f_1 R f_1 \rangle$ . At low energy this ratio is very close to  $\eta$ , reflecting the fact that the transition density at the spurious state energy is close to that of Eq. (15) [19]. At higher energies, this ratio exhibits fluctuations due to the nonnegligible terms  $R_{31}$ , Eq. (10).

Figure 2, exhibits the strength functions for the ISGDR in  $^{208}\text{Pb}$  obtained from Eqs. (5), (4) and (18). The solid line describes the result obtained using  $f_\eta$ . Note that this result coincides with  $S_\eta(E)$ , which is free of SSM contribution. Similarly, the dashed line describes the erroneous result obtained using  $f_3$  (it is also different from  $S_3(E)$ ). We find that when using  $f_3$ , the excitation strengths obtained for certain states are sensitive to the value of  $\Gamma$ . The result obtained with  $f_3$  coincides with that obtained with  $f_\eta$  for  $\Gamma \rightarrow 0$ , as expected since in this

case Eq. (7) has one term. Thus, in configuration RPA calculation of  $\rho_t$ , one may use  $f_3$  and correct for the SSM contribution before the smearing process.

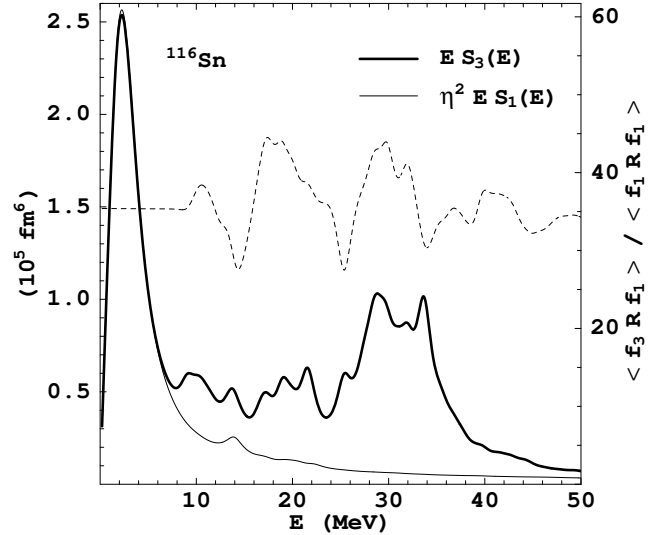


FIG. 1. Energy weighted strength functions for ISGDR in  $^{116}\text{Sn}$ , obtained from Eq. (3) for the scattering operators  $f_3$  (thick line) and  $f_1$  (thin line) with  $\eta = \frac{5}{3}\langle r^2 \rangle = 35.6$  fm $^2$ . Also shown is the ratio  $\langle f_3 R f_1 \rangle / \langle f_1 R f_1 \rangle$  (dashed line).

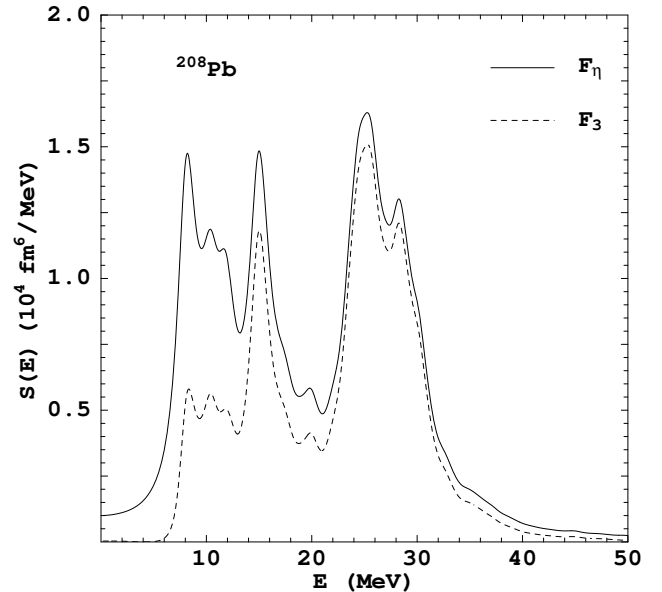


FIG. 2. Strength functions for the ISGDR in  $^{208}\text{Pb}$  obtained from Eqs. (4), (18) and (5), using  $f_3$  (dashed line) and  $f_\eta = f_3 - \eta f_1$  (solid line), with  $\eta = 52.1$  fm $^2$ .

Our results for the ISGDR,  $S_\eta(E)$ , indicate two main components with the low energy component containing close to 30% of the EWSR (for  $E$  up to 23 and 19 MeV for  $^{116}\text{Sn}$  and  $^{208}\text{Pb}$ , respectively), in agreement with the experimental observation [22]. Similar results were obtained for other nuclei and in other calculations [12,19,10]

In Figure 3 we present results of microscopic calcu-

lations of the excitation cross section of the ISGDR in  $^{116}\text{Sn}$  by 240 MeV  $\alpha$ -particle, carried out within the FM-DWBA. The dashed lines are obtained using  $\rho_{coll}(r)$  of the ISGDR [2,17]. It is seen from the upper panel that the use of  $\rho_{coll}$  increases the EWSR by at least 10% and

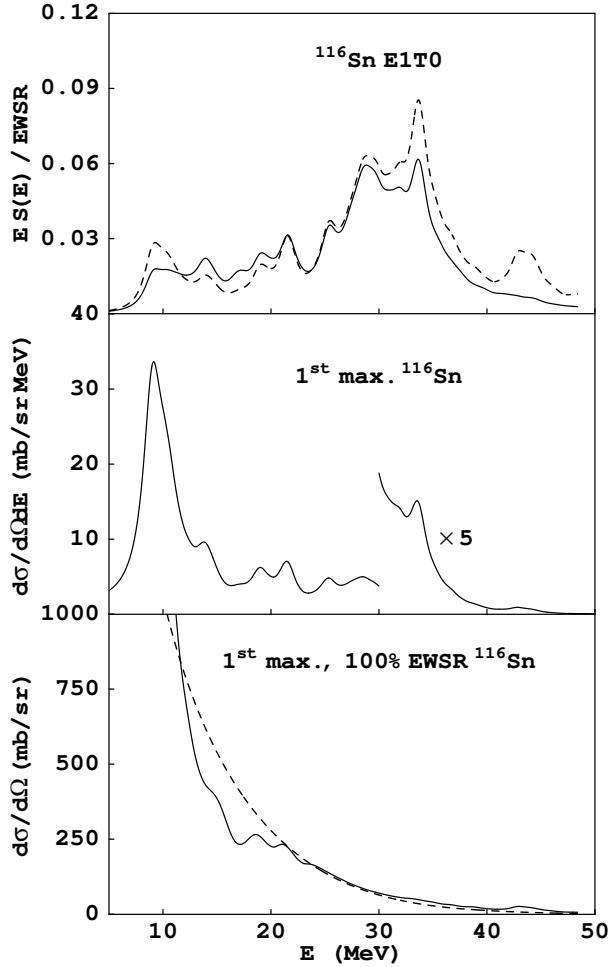


FIG. 3. The ISGDR in  $^{116}\text{Sn}$ . The middle panel: maximum double differential cross section obtained from  $\rho_t$  (RPA). The lower panel: maximum cross section obtained with  $\rho_{coll}$  (dashed line) and  $\rho_t$  (solid line) normalised to 100% of the EWSR. Upper panel: The solid and dashed lines are the ratios of the middle panel curve with the solid and dashed lines of the lower panel, respectively.

may shift the centroid energy by a few percent. An important result of our calculation is that the maximum cross section for the ISGDR drops below the current experimental sensitivity of 2 mb/sr/MeV for excitation energy above 35 MeV (30 MeV for  $^{208}\text{Pb}$ ), which contains about 20% of the EWSR. This missing strength leads to a reduction of more than 2.5 MeV in the ISGDR energy and thus explains the discrepancy between theory and experiment. More sensitive experiments and/or with higher  $\alpha$ -particle energy are thus needed.

In summary, we described and applied an accurate and

general method to eliminate the SSM contributions from  $S(E)$  and  $\rho_t$ . Our results indicate: (i) Existence of non-negligible ISGDR strength at low energy and (ii) Accurate determination of the relation between  $S(E)$  and  $\sigma(E)$  resolves the long standing problem of the conflicting results obtained for  $K$ , deduced from experimental data  $\sigma(E)$  for the ISGDR and data for the ISGMR.

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